Gauge theory non-local operators from two dimensional conformal field theories

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Queen Mary University, October 2010

F.P. arXiv:1003.1151 [hep-th]

C. Kozcaz, S. Pasquetti, F.P. and N. Wyllard, arXiv:1008.1412 [hep-th]

Outline

A large class of 4-dimensional $\mathcal{N} = 2$ gauge theories

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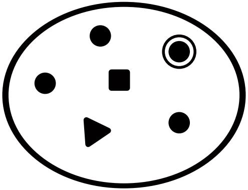
A large class of 4-dimensional $\mathcal{N} = 2$ gauge theories

Gaiotto has constructed a large class of 4-dimensional $\mathcal{N} = 2$ gauge theories that describe the low energy dynamics of a stack of N M5-branes compactified on a punctured Riemann surface $C_{(f_a),g}$ [Gaiotto 09][Witten 97]



A $\mathcal{N} = 2$ gauge theory is associated to any punctured Riemann surface

$$\mathcal{T}_{(f_a),g} \quad \Longleftrightarrow \quad C_{(f_a),g}$$

$\mathcal{T}_{(f_a),g}$ is characterized by the same data labeling the surface

- the genus g
- the number of punctures (f_a). There are different types of puncture and each type is labeled by a Young tableaux with N boxes.

More in details

- (f_a) punctures encode the flavor symmetry of the gauge theory $\mathcal{T}_{(f_a),g}$
- the different degenerations of the surface $C_{(f_a),g}$ such that it becomes a set of pairs of pants connected by thin tubes are associated to the different S-duality frame of the gauge theory $\mathcal{T}_{(f_a),g}$
- the thin tubes connecting the pair of pants are the weakly coupled gauge groups
- the moduli space of $C_{(f_a),g}$ is equal to the physical parameter space of $\mathcal{T}_{(f_a),g}$

Geometrical interpretation of S-duality!!

N=2

Consider $N = 2$, i.e. the low energy theory of 2 M5-branes

- one type of puncture $\Longrightarrow$ only $SU(2)$'s flavor groups
- one type of pair of pants $\Longrightarrow$ only $SU(2)$'s gauge groups

and in this case

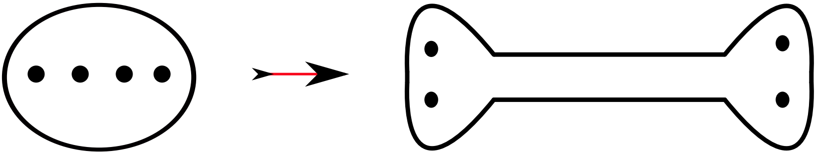
$$C_{f,g} \quad \Longleftrightarrow \quad \mathcal{T}_{f,g}$$

- f punctures $\Longleftrightarrow$ $(SU(2))^f$ flavor group
- $f + 3g - 3$ thin tubes $\Longleftrightarrow$ $(SU(2))^{f+3g-3}$ gauge group

Let's consider a particular example, $C_{4,0}$.

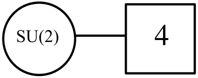
$C_{4,0}$ Riemann surface

Let's consider a degeneration limit of the $C_{4,0}$



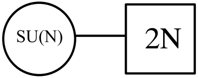
- 4 punctures $\Longleftrightarrow$ $(SU(2))^4$ flavor group
- 1 tube $\Longleftrightarrow$ $SU(2)$ gauge group

This is the flavor and gauge group content of the conformal $\mathcal{N}$=2 $SU(2)$ gauge theory coupled to 4 hypermultiplets ($N_f = 4$)



$\mathcal{N}$=2 $SU(N)$ gauge theory with $N_f = 2N$

What is the Riemann surface associated to the conformal $\mathcal{N}$=2 $SU(N)$ gauge theory coupled to $N_f = 2N$ hypermultiplets ?

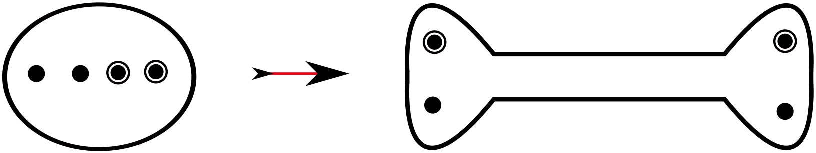


The flavor group is $U(1) \otimes SU(N) \oplus U(1) \otimes SU(N)$. There are two types of flavor group thus we need two types of punctures:

- simple puncture associated to $U(1)$ flavor group
- full puncture associated to $SU(N)$ flavor group

$C_{(2,2),0}$ Riemann surface

The weakly coupled conformal N=2 $SU(N)$ gauge theory coupled to $N_f = 2N$ hypermultiplets is associated to a degeneration of $C_{(2,2),0}$



The tube in the center is associated to the $SU(N)$ gauge group

- It is possible to consider different degenerations of $C_{(2,2),0}$, where the punctures are grouped in different ways
- One of the possible degenerations describes a strongly coupled theory that do not admit a Lagrangian description [AS 07]
- Different theories associate to different degeneration of $C_{(2,2),0}$ are related by the S-duality transformations

Can we use this 2D-4D connection to perform quantitative computations?

[AGT 09]: yes!

From 4D to 2D: the AGT proposal

The partition function of four dimensional gauge theories $\mathcal{T}_{(f_a),g}(A_{N-1})$ defined on S^4 is equivalent to a correlator of two dimensional A_{N-1} Toda field theory defined on $C_{(f_a),g}$ [AGT 09][Wyllard 09]

$$Z_{\mathcal{T}_{f,g}} = \langle V_{m_1} \dots V_{m_f} \rangle_{A_{N-1}\ \textit{Toda on}\ C_{(f_a),g}}$$

- there is one primary for each puncture and the momenta $m_1, \dots, m_f$ are related to the masses of the hypermultiplets
- different correlators of the same 2D field theory compute the partition function of different 4D gauge theories

Where does it come from?

4D Gauge theory side

The partition function for $\mathcal{N} = 2$ gauge theories on S^4 can be written as [Pestun 07]

$$Z_{\mathcal{T}_{f,g}} = \int [da]\ \tilde{Z}^{(\sigma)}\ \bar{\tilde{Z}}^{(\sigma)}$$

- a is the VEV of adjoint scalars in the vector multiplets
- σ labels the S-duality frame

$\tilde{Z}$ includes a perturbative and a non-perturbative factor

$$\tilde{Z} = Z_{\text{pert}} Z_{\text{instanton}}$$

where

$$Z_{\text{instanton}} = Z_{\text{instanton}}(\tau, a, \hat{m}, \epsilon_1, \epsilon_2)$$

- τ is the gauge coupling
- $\hat{m}$ are the hypermultiplets masses
- ϵ_1, ϵ_2 are the deformation parameters

2D CFT side, N=2

A_1 Toda field theory is better known as Liouville field theory. It is described by a 2D Lagrangian $S_{Liou} = S_{Liou}(\phi, b)$

- $\phi(z, \bar{z})$ is a 2D scalar field
- b is dimensionless coupling constant

The theory is conformal invariant. The Virasoro primaries that generate the Hilbert space are given by $V_\alpha(z, \bar{z}) = e^{2\alpha\phi(z,\bar{z})}$, where

- α is the momentum of the primary field
- the conformal dimension is given by $\Delta(\alpha) = \alpha(Q - \alpha)$ where $Q = b + 1/b$

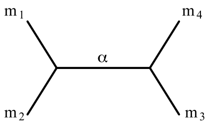
Conformal Bootstrap Approach: any correlator can be expressed in terms of 3-point functions and conformal blocks

E.G. Consider the case of $C_{4,0}$

$$\langle m_1 | V_{m_2}(1) V_{m_3}(z,\bar{z}) | m_4 \rangle$$

$$= \int d\alpha \langle m_1 | V_{m_2}(1) | \alpha \rangle \langle \alpha | V_{m_3}(z,\bar{z}) | m_4 \rangle \mathcal{F}^{(\sigma)}_{\alpha,m_1,m_2,m_3,m_4}(z) \bar{\mathcal{F}}^{(\sigma)}_{\alpha,m_1,m_2,m_3,m_4}(\bar{z})$$

where σ label the decomposition of the conformal block. In this case



- Considering a different decomposition of the correlator the result does not change: modular invariance

AGT have pointed out that

$$Z^{(\sigma)}_{\text{instanton}}(\tau, a, \hat{m}, \epsilon_1, \epsilon_2) = \mathcal{F}^{(\sigma)}_{\alpha,m}(z)$$

$$Z_{\text{pert}} = \text{3-point functions}$$

Considering

- $a \sim \alpha$ VEV of the scalars are related to the internal momenta
- $\hat{m} \sim m$ masses of the hypers are related to external momenta
- $\epsilon_1 = b, \epsilon_2 = 1/b$ deformation parameters are related to the coupling constant
- $e^{2\pi i\tau} = z$ gauge couplings are related to the worldsheet coordinates

S-duality invariance of the partition function follows from modular invariance of the correlator!

2D CFT side, N>2

A_{N-1} Toda field theory is a generalization of Liouville field theory. It is described by a 2D Lagrangian $S_{A_{N-1}} = S_{A_{N-1}}(\phi, b)$

- $\phi = \sum_{k=1}^{N-1} \varphi_k e_k$ where e_k is a simple root of A_{N-1} algebra
- b is a dimensionless coupling constant

There are in total $N-1$ holomorphic currents $W^{(i+1)}$ $(i = 1, \ldots, N-1)$ that realize a $\mathcal{W}_N$ algebra.

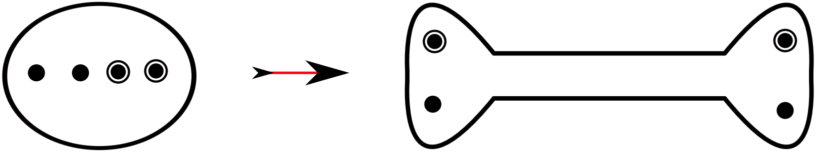
The $\mathcal{W}_N$ primaries that generate the Hilbert space are given by $V_\alpha = e^{\langle \alpha, \phi \rangle}$

- $\langle \cdot, \cdot \rangle$ is the scalar product in the root space
- α is a vector in the root space of the A_{N-1} algebra
- the conformal dimension is given by $\Delta(\alpha) = \frac{1}{2}\langle \alpha, 2Q - \alpha \rangle$

Some special states are

- semidegenerate states. There are null states in the Verma module
- degenerate states. $\alpha = -b\Omega_1 - \frac{1}{b}\Omega_2$, N-1 null states in the Verma module, (maximum number)

Differently from Liouville the theory is not solved, but there are certain correlators that can be computed exactly.
E.G. Consider the case of $C_{(2,2),0}$



Different types of punctures are now associated to different types of primaries

- simple puncture associated to semidegenerate state V_χ with $\chi = \kappa\omega_1$
- full puncture associated to a non-degenerate state V_m with generic m

The correlator

$$\langle m_1 | V_{\chi_2}(1) V_{\chi_3}(z,\bar{z}) | m_4 \rangle$$

$$= \int d\alpha \langle m_1 | V_{\chi_2}(1) | \alpha \rangle \langle \alpha | V_{\chi_3}(z,\bar{z}) | m_4 \rangle \mathcal{F}^{(\sigma)}_{\alpha, m_1, \chi_2, \chi_3, m_4}(z) \bar{\mathcal{F}}^{(\sigma)}_{\alpha, m_1, \chi_2, \chi_3, m_4}(\bar{z})$$

reproduce the partition function for the $\mathcal{N}$=2 $SU(N)$ theory with $N_f = 2N$ hypermultiplets!

What happen if we insert non-local operators in the gauge theory?

What are the operators that we can insert without spoiling the 4D-2D relation?

M-theory picture

The $\mathcal{N} = 2$ 4D theories can be thought as the worldvolume theory of M5-branes compactified on $C_{f,g}$

	4D				C						
	0	1	2	3	4	5	6	7	8	9	10
N M5	X	X	X	X	X	X					

We add to the system other M2 or M5 in order to form supersymmetric intersections

M-theory picture

Let's focus now on this particular M2-M5 intersection

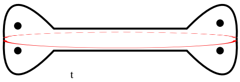
	4D				C						
	0	1	2	3	4	5	6	7	8	9	10
N M5	X	X	X	X	X	X					
M2	X				X		X				

On the M5-branes worldvolume, the M2-brane manifests itself as

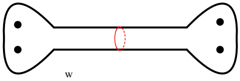
- 1-dimensional operator on the 4D space $\Rightarrow$ $\frac{1}{2}$ BPS Loop Operator!
- 1-dimensional operator on $C_{f,g}$ $\Rightarrow$ Monodromy of a chiral degenerate state [DGOT 09][AGGTV 09]

The monodromy is evaluated along a curve γ on $C_{f,g}$. The magnetic and electric charge of the loop operator are encoded by the curve γ [DMO 09][DGOT 09][AGGTV 09]

- The ’t Hooft Loop is associated to the monodromy along the curve

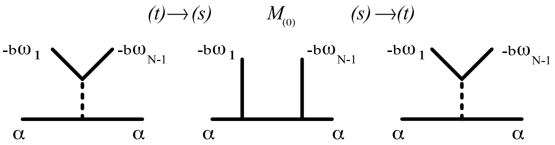


- The Wilson Loop is associated to the monodromy along the curve



The monodromy can be computed inserting the degenerate field and performing a chain of modular transformations of the conformal block

For the Wilson Loop



- For Liouville theory the modular transformations are known explicitly and one can compute any kind of loop operator: electric, magnetic or dyonic. For the Wilson Loop case, the result is in agreement with Pestun [Pestun 07].

- For A_{N-1} Toda field theory the modular transformations are not known explicitly in the general case

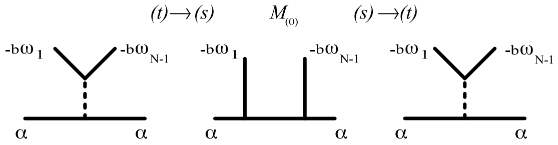
- However, the 4-point correlator relevant for the Wilson Loop involves two degenerate fields and it results

$$\langle V_\alpha(0) V_{-b\omega_1}(z,\bar{z}) V_{-b\omega_{N-1}}(1) V_\alpha(\infty)\rangle = |z|^{2b\langle\alpha,h_1\rangle} |1-z|^{\frac{-2b^2}{N}} G(z,\bar{z})$$

  where $G(z,\bar{z})$ satisfies the generalized hypergeometric equation in the variables z and $\bar{z}$

- The relevant conformal blocks are generalized hypergeometric functions. Modular transformations are linear transformations relating different sets of solutions of the generalized hypergeometric equation

$\hat{\mathcal{L}}$ is the set of modular transformations associated to the Wilson Loop



- $\mathcal{F}^{(s)} = F\,\mathcal{F}^{(t)}$ where F is the fusion matrix

$$\hat{\mathcal{L}} \cdot \mathcal{F}_N^{(t)} = \mathcal{L}\,\mathcal{F}_N^{(t)}$$

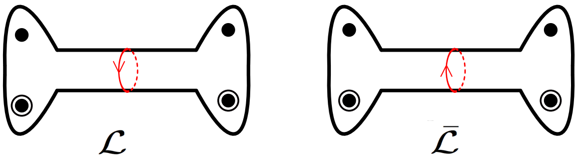
where

$$\mathcal{L} = F^{-1}_{Nk} M^{(s)}_{(0)kr} F_{rN} = M^{(t)}_{(0)NN}$$

- $M^{(s)}_{(0)}$ monodromy matrix respect to 0 for conformal blocks $\mathcal{F}^{(s)}$
- $M^{(t)}_{(0)}$ monodromy matrix respect to 0 for conformal blocks $\mathcal{F}^{(t)}$

$\mathcal{L}$ can be computed using properties of the monodromy group of the generalized hypergeometric equation. It results [Passerini 10]

- the Wilson Loop in the fundamental and antifundamental representation is associated to the monodromy of the chiral degenerate field $V_{-b\omega_1}$
- the orientation of the curve selects between fundamental and antifundamental representation
- the orientation of the monodromy curve is a new feature of the $N > 2$ case



- The correlator associated to the partition function of the conformal $SU(N)$ with $N_f = 2N$

$$\langle m_1 | V_{\chi_2}(1) V_{\chi_3}(z,\bar{z}) | m_4 \rangle$$

- The Wilson Loop monodromy modifies the correlator as

$$\int d\alpha \mathcal{L} \langle m_1 | V_{\chi_2}(1) | \alpha \rangle \langle \alpha | V_{\chi_3}(z,\bar{z}) | m_4 \rangle \mathcal{F}^{(\sigma)}_{\alpha, m_1, \chi_2, \chi_3, m_4}(z) \bar{\mathcal{F}}^{(\sigma)}_{\alpha, m_1, \chi_2, \chi_3, m_4}(\bar{z})$$

where

$$\mathcal{L} = \frac{1}{N} \mathrm{Tr}_F\, e^{i2\pi ba} \qquad \bar{\mathcal{L}} = \frac{1}{N} \mathrm{Tr}_{\bar{F}}\, e^{i2\pi ba}$$

- Using the AGT dictionary, this result is in agreement with the Wilson Loop expectation value obtained by Pestun [Pestun 07] using gauge theory! In agreement also with [DGG 10]

M-theory picture

Let's now consider the following M5-M5 intersection

	4D				C						
	0	1	2	3	4	5	6	7	8	9	10
N M5	X	X	X	X	X	X					
M5	X	X			X	X	X	X			

On the N M5-branes worldvolume, the intersecting M5-brane manifests itself as

- 2-dimensional operator on the 4D space $\Rightarrow$ $\frac{1}{2}$ BPS Surface Operator!
- wrap completely the $C_{f,g}$ $\Rightarrow$ different 2-dimensional CFT! [AT 10]

As argued by Alday and Tachikawa [AT 10], this M-theory setup describes a full ramified surface operator [GW 06]

full ramified surface operators

A ramified surface operator is defined imposing that the fields in the theory posses a certain singularity on the 2D subspace where the operator is supported

On the plane where a full operator is located

- the unbroken gauge group is $U(1)^{N-1}$
- $N-1$ monopole numbers $\ell_i = \frac{1}{2\pi}\int_{z_2=0} F_i$

A full ramified instanton is characterized by N topological quantities $(k, \ell_1, \ldots, \ell_{N-1})$

Instanton function with surface operators

The instanton partition function is given by [Braverman 04][BE 04][Negut 08][FFNR 08][AT 10]

$$Z_{instanton} = \sum_{\lambda} Z_{\vec{k}}(\lambda) \prod_{i} y_i^{k_i}$$

- $k_1 = k \qquad k_{i+1} = k_i + \ell_i$
- $\lambda = (\lambda_1, \ldots, \lambda_N)$ is a vector of Young tableau
- $k_i = \sum_{j \geq 1} \lambda_j^{i-j+1}$
- $Z_{\vec{k}}(\lambda)$ depends on the field content

$\mathcal{N}=2$ $SU(N)$ gauge theory with $N_f=2N$

Let's focus on the $\mathcal{N}=2$ $SU(N)$ gauge theory with $N_f=2N$ hypers with a full ramified surface operator. We define [KPPW 10]

- $Z^{(0),i}$ the sum of all terms with $k_i \neq 0$ and $k_j=0$ for $i\neq j$
- $Z^{(1),i,j}$ the sum of all terms with $k_i\neq 0$, $k_j=1$ for and $k_r=0$ for $r\neq i,j$

Thus

$$Z_{instanton}=\sum_i Z^{(0),i}+\sum_{i,j} Z^{(1),i,j}\dots$$

where

$$Z^{(0),i}=\sum_{n=1}^{\infty}\frac{(\frac{\mu_{i+1}}{\epsilon_1}-\frac{a_i}{\epsilon_1}+\frac{\epsilon_2}{\epsilon_1}\lfloor\frac{i}{N}\rfloor+1)_n(\frac{\tilde{\mu}_i}{\epsilon_1}-\frac{a_i}{\epsilon_1})_n}{(\frac{a_{i+1}}{\epsilon_1}-\frac{a_i}{\epsilon_1}+\frac{\epsilon_2}{\epsilon_1}\lfloor\frac{i}{N}\rfloor+1)_n\, n!}(-y_i)^n$$

- a_i are the Coulomb branch parameters
- $\mu_i, \tilde{\mu}_i$ are the hypermultiplets masses

$\mathcal{N}=2$ $SU(2)$ theories and affine $SL(2)$ algebra

The instanton partition function for $\mathcal{N}=2$ $SU(2)$ gauge theories with a full ramified surface operator are equivalent to modified affine $SL(2)$ conformal blocks [AT 10]

Affine $SL(2)$ algebra is defined by

$$[J_n^0, J_m^0] = \frac{k}{2}\, n\, \delta_{n+m,0}\,, \quad [J_n^0, J_m^\pm] = \pm J_{n+m}^\pm\,, \quad [J_n^+, J_m^-] = 2J_{n+m}^0 + k\, n\, \delta_{n+m,0}$$

- $|j\rangle$ primary state, $J_0^0|j\rangle = j|j\rangle$ and $J_{1+n}^-|j\rangle = J_{1+n}^0|j\rangle = J_n^+|j\rangle = 0$
- $V_j(x,z)$ primary field, x is an isospin variable and z is the worldsheet coordinate

The generators act on the primary fields as differential operators

$$[J_n^A, V_j(x,z)] = z^n D^A V_j(x,z)$$

$$D^+ = 2j\,x - x^2\partial_x\,, \qquad D^0 = -x\partial_x + j\,, \qquad D^- = \partial_x$$

E.G. for the $\mathcal{N} = 2$ $SU(2)$ gauge theory with $N_f = 4$ hypermultiples it results

$$Z_{\text{instanton}} = (1 - z)^{2j_2(-j_3+k/2)} \langle j_1 | \mathcal{V}_{j_2}(1,1) \mathcal{V}_{j_3}(x,z) | j_4 \rangle$$

where

$$\mathcal{V}_j = \mathcal{K} V_j \qquad \mathcal{K}(x,z) = \exp\left[-\sum_{n=1}^{\infty} \frac{1}{2n-1} \left(z^{n-1} x\, J^-_{1-n} + \frac{z^n}{x} J^+_{-n} \right) \right]$$

It can be evaluated pertubatively considering the decomposition

$$\sum_{\mathbf{n},\mathbf{A};\mathbf{n}',\mathbf{A}'} \langle j_1 | \mathcal{V}_{j_2}(1,1) | \mathbf{n}, \mathbf{A}; j \rangle X^{-1}_{\mathbf{n},\mathbf{A};\mathbf{n}',\mathbf{A}'}(j) \langle \mathbf{n}', \mathbf{A}'; j | \mathcal{V}_{j_3}(x,z) | j_4 \rangle$$

The components $Z^{(0),i}$ of the instanton function are reproduced by terms with the following internal states [KPPW 10]

- $(J^-_0)^n |j\rangle$ that gives a x^n term
- $(J^+_{-1})^n |j\rangle$ that gives a $\left(\frac{z}{x}\right)^n$ term

The complete dictionary is

- $y_1 = x\,, \qquad y_2 = \frac{z}{x}\,, \qquad j = -\frac{1}{2} + \frac{a_1}{\epsilon_1}\,, \quad k = -2 - \frac{\epsilon_2}{\epsilon_1}$
- $j_1 = -\frac{\epsilon_1+\epsilon_2+\mu_1-\mu_2}{2\epsilon_1}\,, \qquad j_2 = -\frac{2\epsilon_1+\epsilon_2+\mu_1+\mu_2}{2\epsilon_1}$
- $j_3 = -\frac{2\epsilon_1+\epsilon_2-\tilde{\mu}_1-\tilde{\mu}_2}{2\epsilon_1}\,, \quad j_4 = -\frac{\epsilon_1+\epsilon_2+\tilde{\mu}_1-\tilde{\mu}_2}{2\epsilon_1}$

What about the $SU(N)$ gauge theories with N>2 ? [KPPW 10]

$\mathcal{N} = 2$ $SU(N)$ theories and affine $SL(N)$ algebra

Affine $SL(N)$ algebra is generated by

$$J^i_n\,, \qquad J^{i+}_n\,, \qquad J^{i-}_n\,, \qquad J^{il}_n \quad (i \neq l)$$

- $|j\rangle$ primary state, is labeled by $j = \sum_{i=1}^{N-1} j^i \omega_i$ where ω_i are the fundamental weights of $SL(N)$
- $J^i_0|j\rangle = j^i|j\rangle$ and $J^{i+}_0|j\rangle = 0\,, \quad J^{il}_0|j\rangle = 0 \ \ (i > l)\,, \quad J^A_n|j\rangle = 0 \ \ (n > 0)$
- $V_j(x, z)$ primary field, x is a vector of isospin variables and z is the worldsheet coordinate

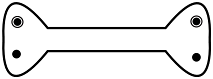
The generators act on the primary fields as differential operators

$$[J^A_n, V_j(x, z)] = z^n D^A V_j(x, z)$$

In general, x is a vector of $\frac{N(N-1)}{2}$ isospin variables and D^A are differential operators in these variables. Too many isospin variables!

The primary field V_χ with $\chi = \kappa\omega_1$ depends only on $N-1$ isospin variables and the action of the generators on these fields is expressed in terms of differential operators D^A that depends on $N-1$ isospin variables

Let's focus on the conformal $\mathcal{N}$=2 $SU(N)$ gauge theory coupled to $N_f = 2N$ hypermultiplets, i.e.



- simple puncture associated to a state V_χ
- full puncture associated to a state V_j with generic j

The instanton function is equivalent to

$$\sum_{\mathbf{n},\mathbf{A};\mathbf{n}',\mathbf{A}'} \langle j_1|\mathcal{V}_{\chi_2}(1,1)|\mathbf{n},\mathbf{A};j\rangle X^{-1}_{\mathbf{n},\mathbf{A};\mathbf{n}',\mathbf{A}'}(j)\langle \mathbf{n}',\mathbf{A}';j|\mathcal{V}_{\chi_3}(x,z)|j_4\rangle$$

where

$$\mathcal{V}_{\chi_i}(x,z) = V_{\chi_i}(x,z)\mathcal{K}^\dagger(x,z)$$

and the dictionary is

- $y_1 = x_1\,, \qquad y_{i+1} = \frac{x_{i+1}}{x_i} \quad (1 \le i \le N-2)\,, \qquad y_N = \frac{z}{x_{N-1}}$
- $j^i = -\frac{1}{2} + \frac{a_i - a_{i+1}}{2\epsilon_1}\,, \qquad k = -N - \frac{\epsilon_2}{\epsilon_1}$
- $\frac{\tilde{\mu}_i}{2\epsilon_1} = -\frac{\kappa_3}{N} + \langle h_i, j_4 + \frac{\rho}{2}\rangle\,, \qquad \frac{\mu_i}{2\epsilon_1} = \frac{\kappa_2}{N} + \langle h_i, j_1 + \frac{\rho}{2}\rangle$

Conclusions

- We have shown that also also when $N > 2$ the AGT can be extended to provide a 2D description of 4D gauge theory Wilson loops.
- We have shown that orientation of the monodromy is relevant for the charge of the loop, when $N > 2$.
- Can we classify loop operators when $N > 2$? [GL 10]
- We extended the AT proposal to the case $N > 2$ and to the non-conformal theories.
- Can we reproduce the full partition function considering the correlator of some CFT with affine algebra?
- What is the physical meaning of the $\mathcal{K}$ operator?