Gauge theory, line operators and dualities

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CERN Theory Group Retreat

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My Interests:

- formal aspects of supersymmetry
- non-local operators in gauge theory
- interconnections between gauge theory, string theory (or M-theory), 2d CFT and matrix models

AdS/CFT: The strong coupling regime of a gauge theory is encoded by the dynamics of weakly coupled strings in a certain background.

$SU(N)$ $\mathcal{N}$=4 SYM: the dual string theory is IIB on $AdS_5 \times S^5$ with tension $T = \frac{\sqrt{\lambda}}{2\pi}$, where $\lambda = Ng^2$. The conformal 4d gauge theory lives on the boundary of AdS_5

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SUSY Wilson Loops

$$W_R(C) = \frac{1}{N} \, \mathrm{tr}\,_R \mathcal{P} \exp \left[\int_C ds \, \left(i A_\mu(x) \dot{x}^\mu + n_I \Phi_I(x) |\dot{x}| \right) \right]$$

In the string theory dual, the Wilson loop is associated to an open string that ends on the boundary of AdS_5, on the loop where the operator is supported [Maldacena][Rey, Yee]

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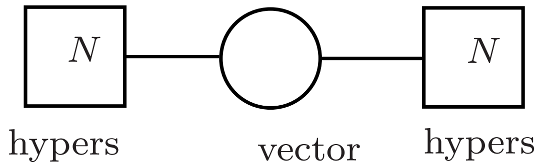
Strong coupling $\lambda \gg 1$ and $N \to \infty$, semiclassical string [Drukker,Gross]

$$\langle W(C_{\text{circle}}) \rangle_{\mathcal{N}=4} \simeq K T^{-3/2} e^{2\pi\, T}$$

In agreement with an exact gauge theory computation using localization [Pestun]

Next Step: reduce the symmetry $\Rightarrow$ increase complexity of the theory

$\mathcal{N} = 2$ $SU(N)$ SCYM:



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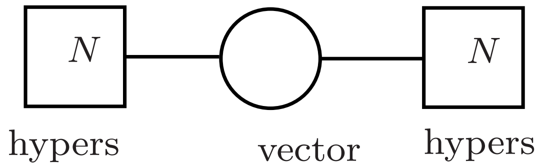

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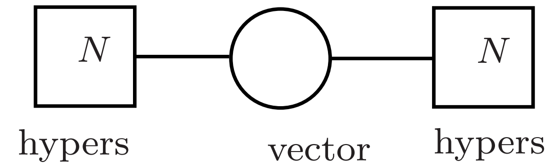

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Pestun Localization: reduce the field theory path integral to a matrix integral, for any value of the gauge coupling. VEV of certain non-local operators can be computed using a matrix model.

$$Z = \int D\Psi e^{-S[\Psi]} = \int DM e^{-S[M]} Z_{\text{1-loop}}(M) Z_{\text{inst}}(M)$$

- M is a constant value of an adjoint scalar, i.e. a matrix

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Strong coupling $\lambda \gg 1$, Large N limit: $N \to \infty$ with $\lambda = Ng^2$ fixed [FP, Zarembo]

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that is equivalent to the semiclassical string in AdS_5

$$\langle W(C_{\text{circle}})\rangle_{\lambda\gg 1} = KT^{-3/2}e^{2\pi T}$$

considering the string tension

$$T = \frac{3}{2\pi}\ln\lambda$$

AGT correspondence: The partition function of conformal $\mathcal{N}=2$ theories on S^4 with gauge group $SU(N)$ is equivalent to a correlation function in A_{N-1} Toda CFT.
(A_1 Toda = Liouville) [Alday, Gaiotto, Tachikawa]

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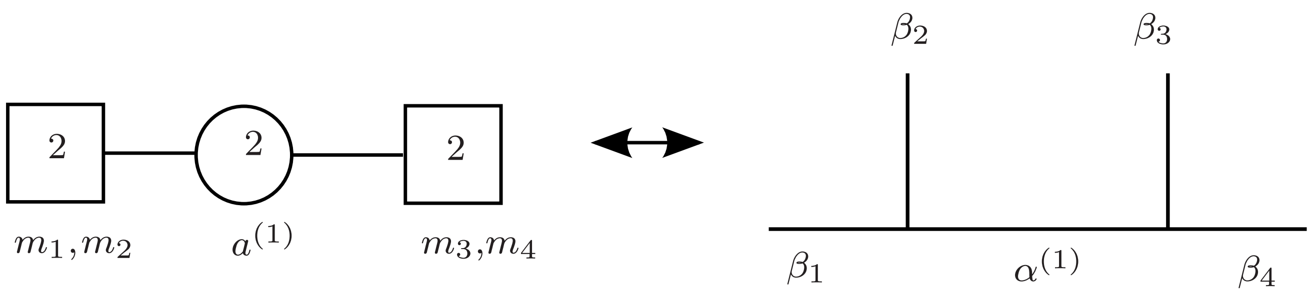
Basic example: $SU(2)$ SCYM

$$Z_{SCYM} = \langle V_{\beta_1} V_{\beta_2} V_{\beta_3} V_{\beta_4} \rangle_{\text{Liouville}}$$



$$\int da^{(1)}\ Z_{\text{cl}}\ Z_{\text{1-loop}}\ Z_{\text{inst}} = \int d\alpha^{(1)} \langle \beta_1 | V_{\beta_2} | \alpha^{(1)} \rangle \langle \alpha^{(1)} | V_{\beta_3} | \beta_4 \rangle \mathcal{F}_{\alpha,\beta}(z) \bar{\mathcal{F}}_{\alpha,\beta}(\bar{z})\, |z|^{2(\Delta_\alpha - \Delta_{\beta_3} - \Delta_{\beta_4})}$$

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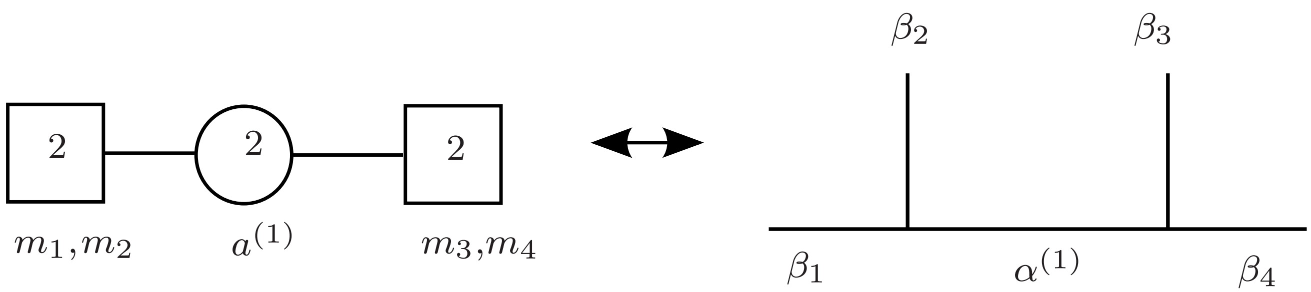

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- Different S-duality frames of the gauge theory correspond to the different decomposition of the 2d CFT correlator
- modular invariance of 2d CFT implies S-duality invariance of the 4d partition function

- Wilson, 't Hooft and dyonic line operators are realized as loop operators in Liouville/Toda CFT (Verlinde operators)
- modular transformations of loop operators in Toda CFT describe the action of S-duality on line operators in gauge theory
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3d SUSY gauge theories have many interesting dualities: Mirror symmetry, AdS/CFT ...

- two main types of line operators: Wilson loops and Vortex loops
- interesting to study the vev of these operators using localization and analyze their role in dualities [Drukker, FP, Okuda, To appear.]

Conclusions

- supersymmetry provides a simplified framework to study Quantum Field Theory
- many novel tools for the weak and strong coupling dynamics of quantum fields (AdS/CFT, Pestun Localization, AGT, Integrability, . . .)
- interconnections between gauge theory, string theory, two dimensional CFT and matrix models are useful to study the strong coupling dynamics of gauge theory and more formal aspects of string theory