On Holographic Non-Local Operators and Multiple M2-Branes Theories

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Outline of the Thesis

- Holographic Non-Local Operators
  - Wilson Loops
  - Surface Operators
- Multiple M2-branes Theories
  - Non-perturbative formulation of Type IIB string
  - M2-Branes physics from symmetry algebra of BL-Theory

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Holographic Wilson Loops

- Motivations
- Wilson Loops: definition and basic properties
- AdS\CFT duality
- Holographic description of Wilson loop operators

Motivations

- Wilson Loops: non-local and gauge invariant operators

  -Insert external electrically charged particles in gauge theories

  -Can be used as order parameter to study the phase structure of gauge theories

  -Gauge theories can be formulated in terms of Wilson loop variables

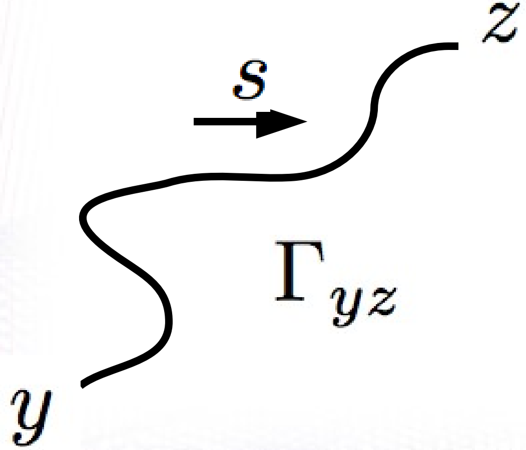
→ Given their importance for gauge theories, it is interesting to study these operators in the context of the AdS/CFT duality

Wilson Loops

- $U(1)$ Gauge Theory $\quad [\ A_\mu(x)\ ]$

  Describes the dynamics of the electromagnetic fields

- *Consider a charged particle propagating along* Γ_{yz}



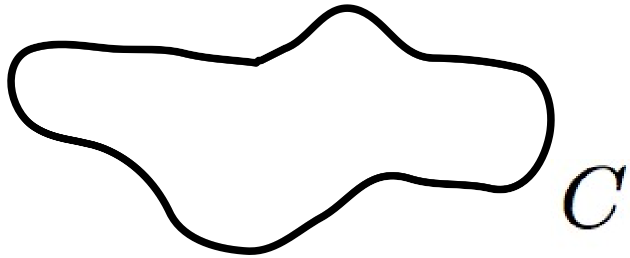
$$S_{int} = q \int_{\Gamma_{yz}} ds\, \dot{x}^\mu A_\mu$$

Wilson Loops

- *The phase factor associated to the charged particle*

$$V[\Gamma_{yz}] = \exp\left(iq \int_{\Gamma_{yz}} ds\, \dot{x}^\mu A_\mu \right)$$

- *Considering* $\Gamma = C$



The phase factor is invariant under gauge transformation

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \alpha(x)$$

Wilson Loops

$$W_q(C) = \exp\left(iq \int_C ds\, \dot{x}^\mu A_\mu\right)$$

- Describe the coupling of the gauge theory with a non-dynamical external particle

- *Characterized by*
  - q Charge [Representation of $U(1)$]
  - C Closed Curve

Wilson Loops

- $U(N)$ Gauge Theory [$A_\mu = A_\mu^a t^a$]

$$W_R(C) = \mathrm{Tr}_R\, P \exp\left(i \int_C ds\, \dot{x}^\mu A_\mu \right)$$

- $W_R(C)$ *is Characterized by:*
  - C Closed Curve
  - R Representation of $U(N)$

- *Describe the coupling of an external particle with charge* R *to the* $U(N)$ *Yang Mills theory*

Wilson Loops

- $R = (n_1, n_2, \ldots, n_i, \ldots, n_N)$ is summarized by a Young Tableau

1	2	$\cdot$	$\cdot$	$\cdot$	$\cdot$	n_1
1	2	$\cdot$	$\cdot$	$\cdot$	n_2	
1	2	$\cdot$	$\cdot$	$\cdot$	n_3	
$\cdot$	$\cdot$	$\cdot$	$\cdot$			
1	2	$\cdot$	n_N			

$$n_1 \geq n_2 \geq \ldots \geq n_N \geq 0$$

Wilson Loops

- $U^i{}_j$ is a $N \times N$ matrix

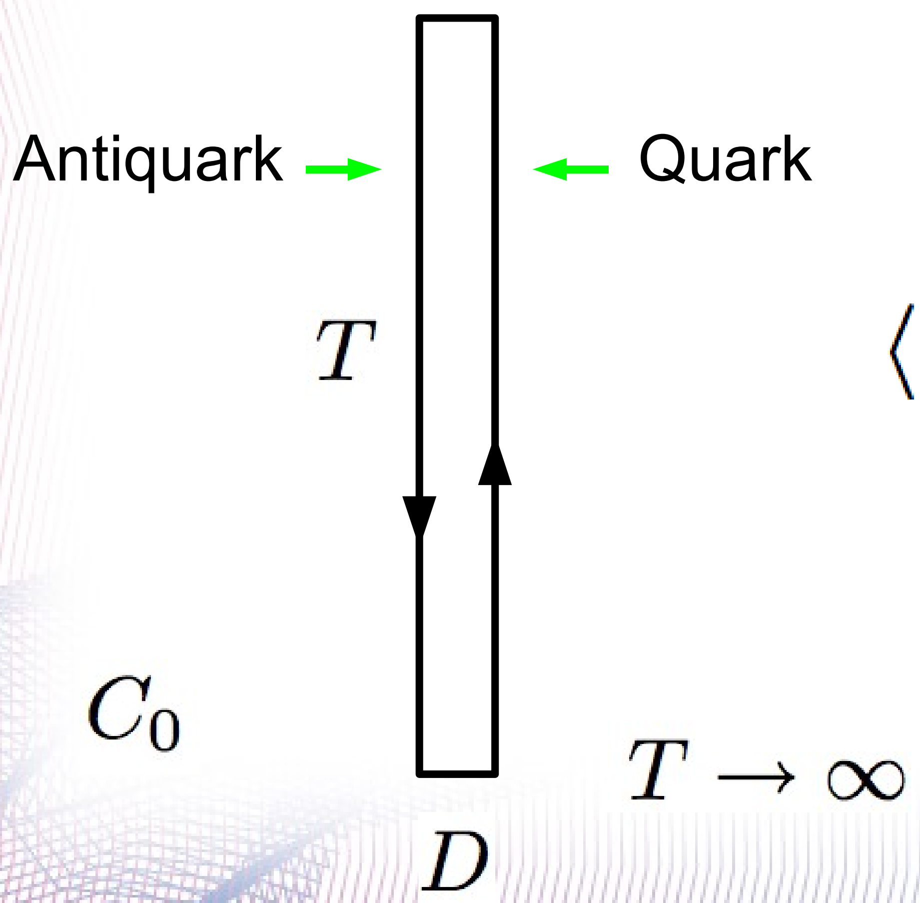
- $$\mathrm{Tr}\, U = \sum_{i=1}^{N} U^i{}_i$$

- $$\mathrm{Tr}_{\square\square}\, U = \sum_{i,j=1}^{N} U^i{}_{(i} U^j{}_{j)} = \frac{1}{2}\left((\mathrm{Tr}\, U)^2 + \mathrm{Tr}\, U^2\right)$$

- $$\mathrm{Tr}_{\substack{\square\\\square}}\, U = \sum_{i,j=1}^{N} U^i{}_{[i} U^j{}_{j]} = \frac{1}{2}\left((\mathrm{Tr}\, U)^2 - \mathrm{Tr}\, U^2\right)$$

Wilson Loops

- If the external particle is a quark , $R = \square$
- Considering the path C_0



$$\langle W_{\square}(C_0) \rangle \sim e^{-TE(D)}$$

Quark-Antiquark potential

Wilson Loops

The Quark-Antiquark potential characterizes the phases of gauge theories

- $E(D) \sim \sigma D$ confining phase
- $E(D) \sim \frac{1}{D}$ Coulomb phase
- $E(D) \sim constant$ Higgs phase

AdS/CFT Duality

Type IIB string

on $\mathrm{AdS}_5 \times \mathrm{S}^5$

with $\int_{S^5} F_5 = N$

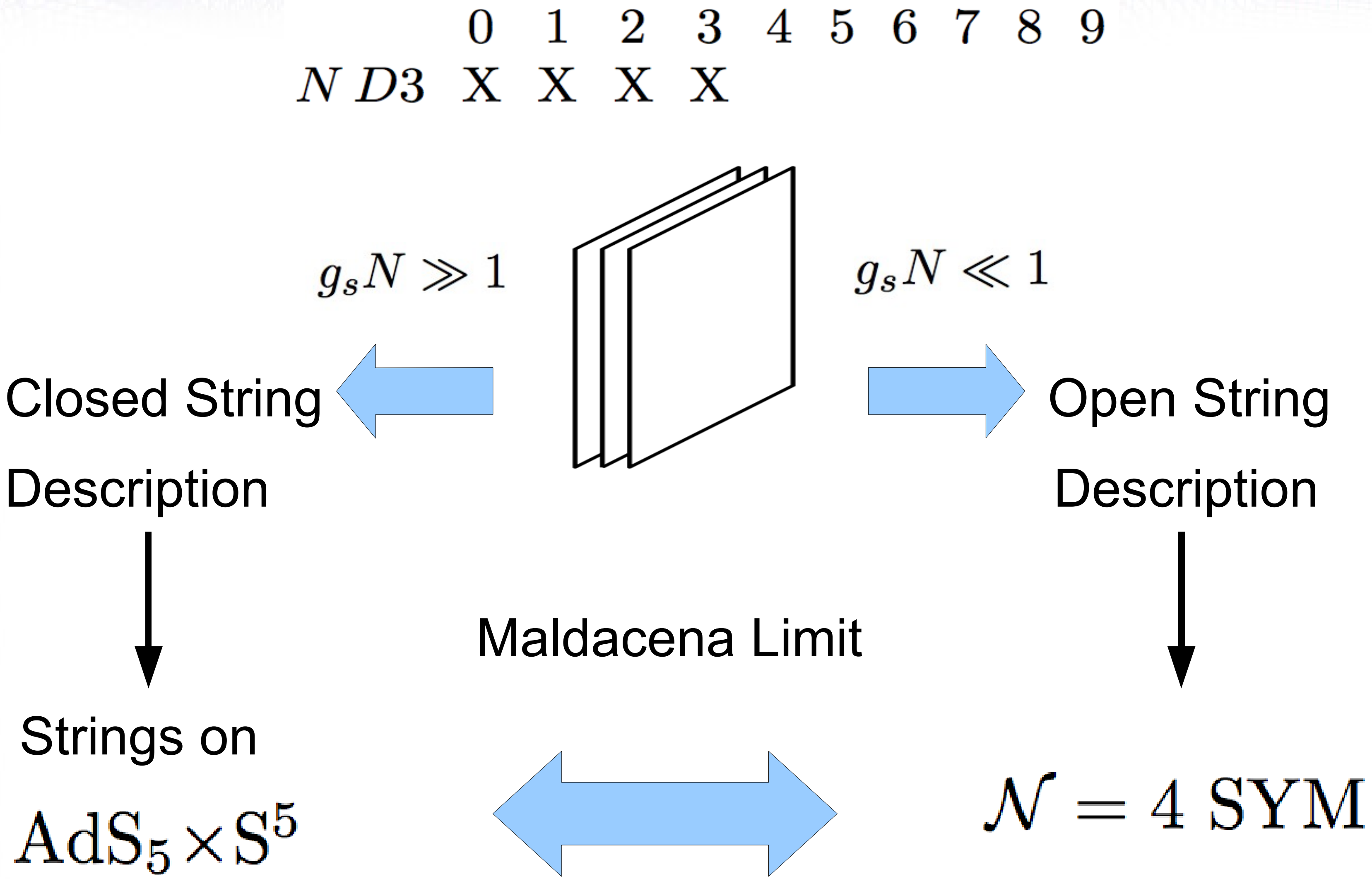
⟺

$\mathcal{N} = 4$ SYM

gauge group $U(N)$

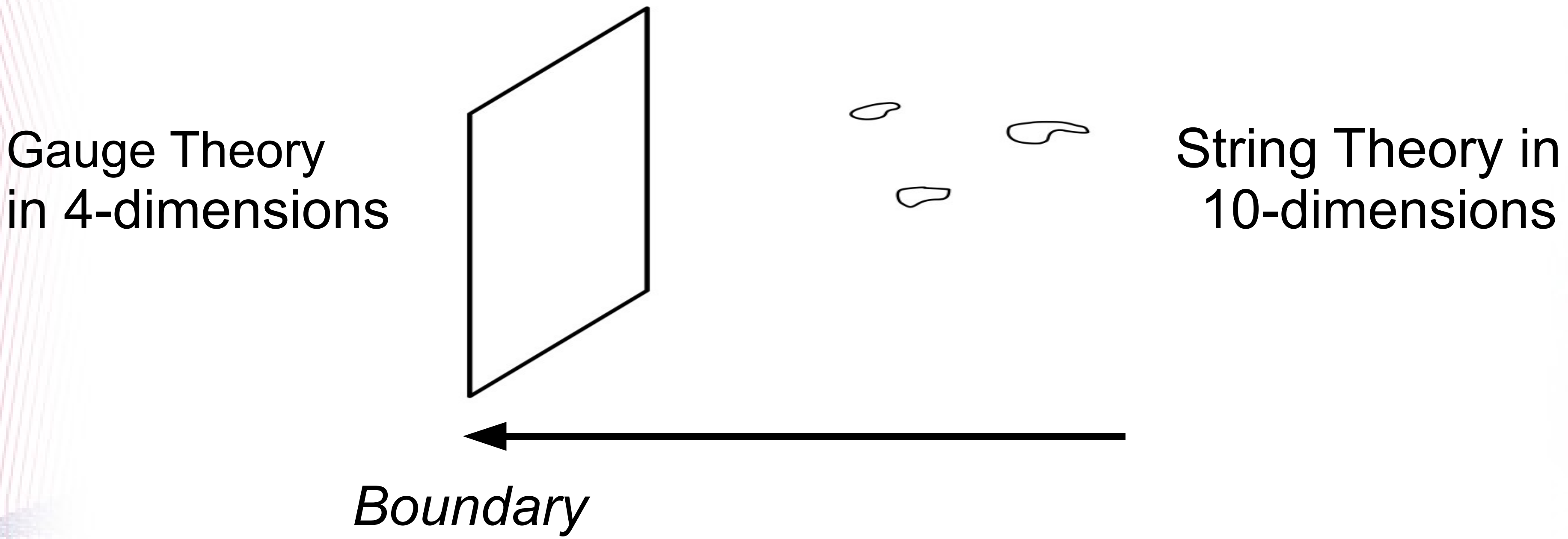
(3+1)-dimension

Theory of Gravity ⟺ *Quantum Field Theory*


0 1 2 3 4 5 6 7 8 9
$N\ D3$ X X X X
$g_s N \gg 1$
$g_s N \ll 1$
Closed String Description
Open String Description
Maldacena Limit
Strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$
$\mathcal{N} = 4$ SYM

Holography

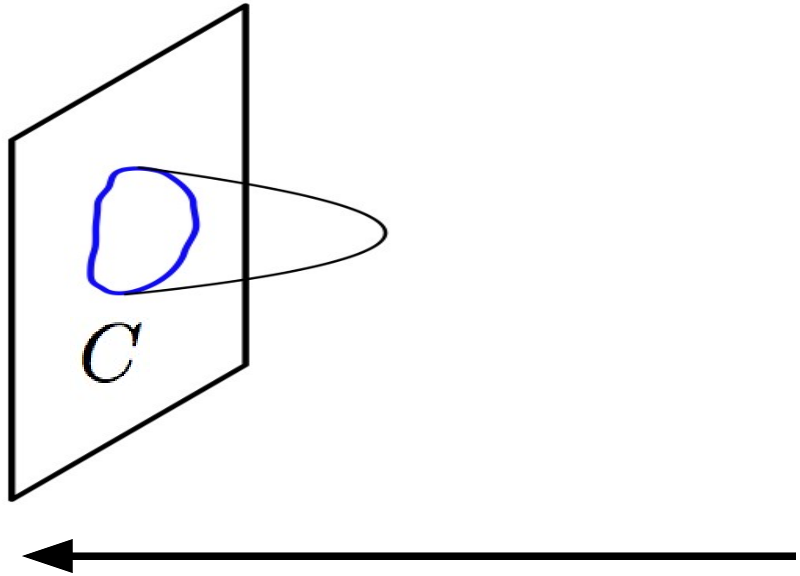
- The 10-dimensional spacetime where the string theory lives posses a 4-dimensional boundary
- The gauge theory is defined on the 4-dimensional boundary



- The physics of the bulk results from a holographic image of the physics on the boundary

Holography

- $W_{\square}(C)$ is associated to a string in $\mathrm{AdS}_5 \times \mathrm{S}^5$ ending on C at the boundary of AdS



- *at strong coupling*

$$\langle W_{\square}(C) \rangle \sim e^{-S_{string}}$$

String action

Holography

Fundamental charges ⟷ Strings

Higher representation charges ⟷ ????????

SYM Wilson Loops

- $U(N)\ \ \mathcal{N}=4\,\mathrm{SYM}$ $\qquad [\ A_\mu, \Phi^I, \Psi\ ]$

$$W_R(C) = \mathrm{Tr}_R\, P\exp\left(i\int_C ds(A_\mu \dot{x}^\mu + \phi_I \dot{y}^I)\right)$$

- *W-L* describe the coupling of a particle with charge R to the SYM

SYM Wilson Loops

- We focus on *Half-BPS W-L*

$$W_R = W_{(n_1,n_2,\dots,n_N)} = \mathrm{Tr}_R\, P \exp\left(i \int dt\, (A_0 + \phi) \right)$$

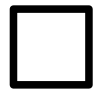
Defined on a line or a circle

- *Characterized by:* R *representation of* $U(N)$

Half-BPS W-L ↔

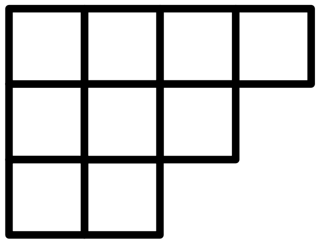
1	2	·	·	·	·	n_1
1	2	·	·	·	n_2	
1	2	·	·	·	n_3	
·	·	·	·			
1	2	·	n_N			

Higher Representations



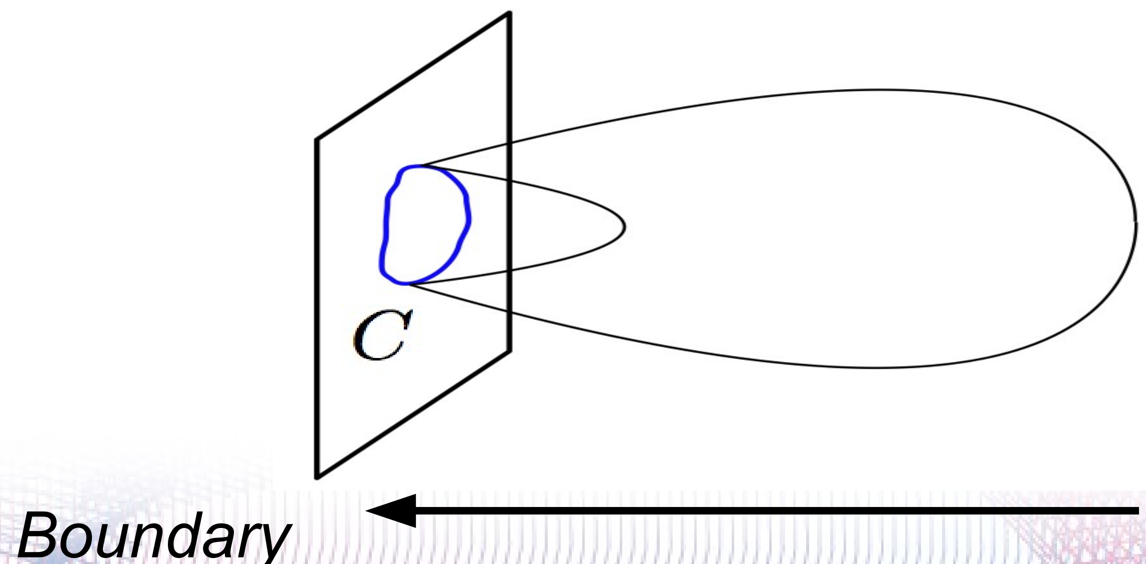
↔ Fundamental String

k Boxes



↔ Multiple k Coincident Fundamental Strings

- Multiple strings can be described in terms of a single D-brane with k units of fundamental string charge



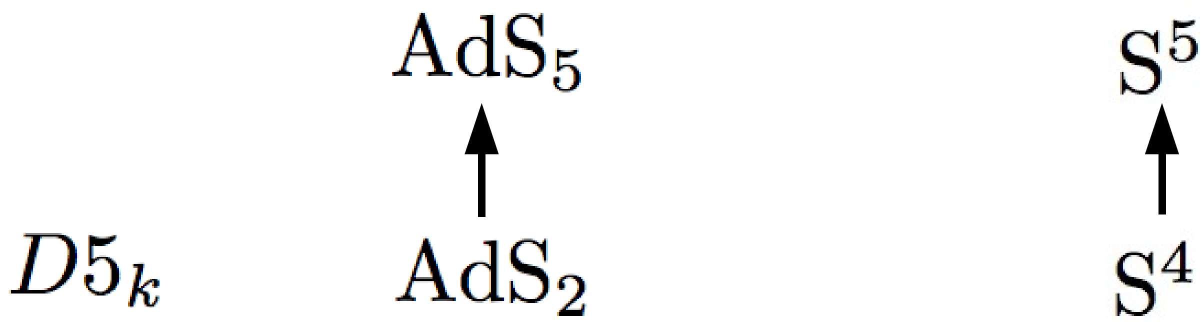
Higher Representations

Look for D-brane probes in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ space such that:

- Preserve the same symmetry as the Half-BPS Wilson loops
- End on the curve where the Wilson loops are defined, at the boundary of AdS

Giant Wilson Loops

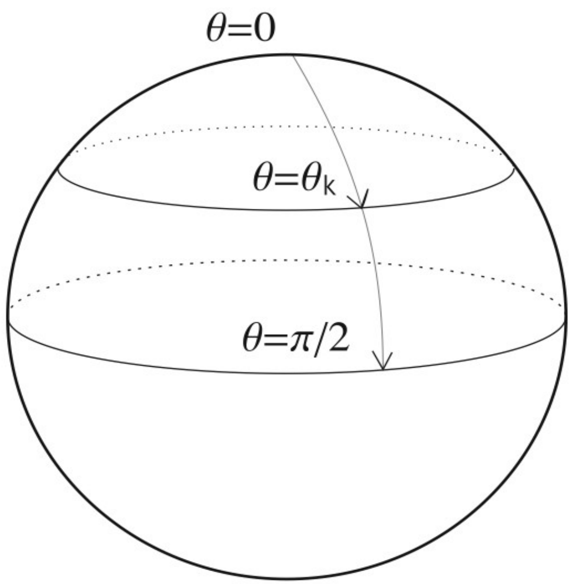
- $D5_k$ has k units of string charge and $\mathrm{AdS}_2 \times \mathrm{S}^4$ worldvolume



- The symmetry preserved is the same as the Half-BPS W-L
- At the boundary of AdS, the $D5_k$ ends on the time line

Giant Wilson Loops

- Embedding in S^5



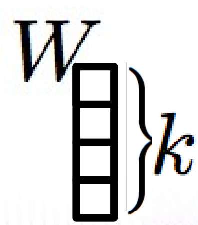
θ_k latitude of the S^4

θ_k increase with k

$\theta_0 = 0 \qquad \theta_N = \pi$

⇒ k cannot be larger than N

$D5_k$ ⇔



Dual Giant Wilson Loops

- $D3_k$ has k units of string charge and $\mathrm{AdS}_2 \times \mathrm{S}^2$ worldvolume

AdS_5 $\quad\quad$ S^5

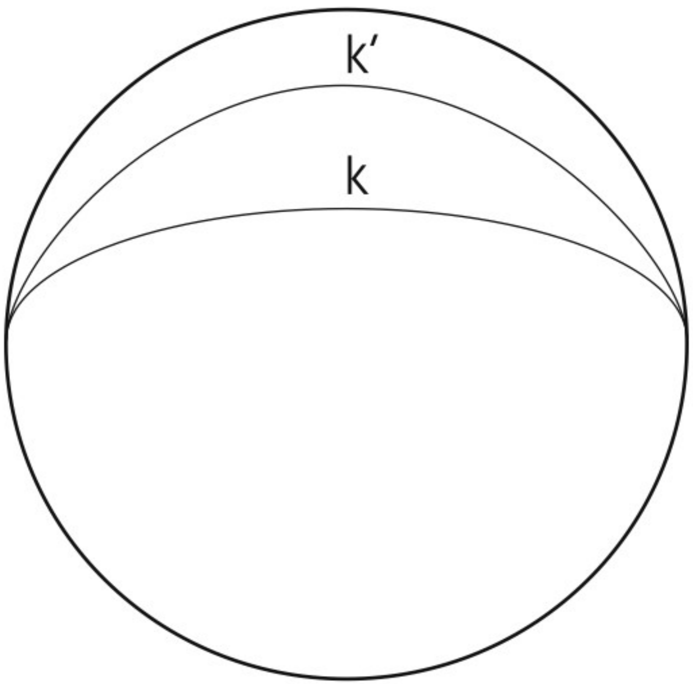
↑ $\quad\quad$ ↑

$D3_k$ $\quad$ $\mathrm{AdS}_2 \times \mathrm{S}^2$ $\quad\quad$ Point

- The symmetry preserved is the same as the Half-BPS W-L
- At the boundary of AdS, the $D3_k$ ends on the time line

Dual Giant Wilson Loops

- Embedding in AdS_5



$\mathrm{AdS}_2 \times \mathrm{S}^2$ foliation of AdS_5

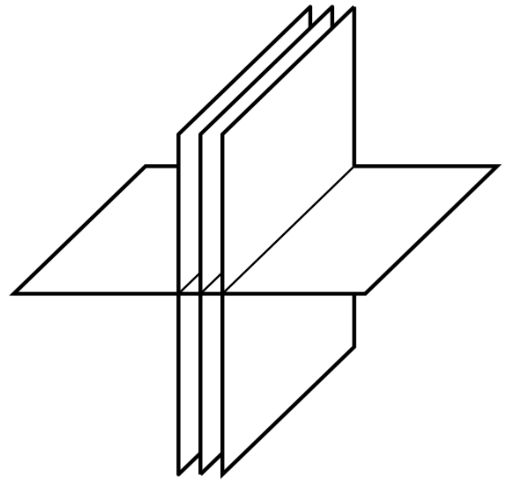
k determines the slice

⇒ k is not bounded

$$D3_k \Longleftrightarrow W_{\underbrace{\square\square\square\square}_{k}}$$

	0	1	2	3	4	5	6	7	8	9
$N\ D3$	X	X	X	X						
$D5$	X					X	X	X	X	X
$k\ F1$	X				X					

$g_s N \gg 1$ ← Closed String Description



Maldacena Limit

$g_s N \ll 1$ → Open String Description

Strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

$D5_k$ probe

$\mathcal{N} = 4$ SYM

1-dimensional defect field theory

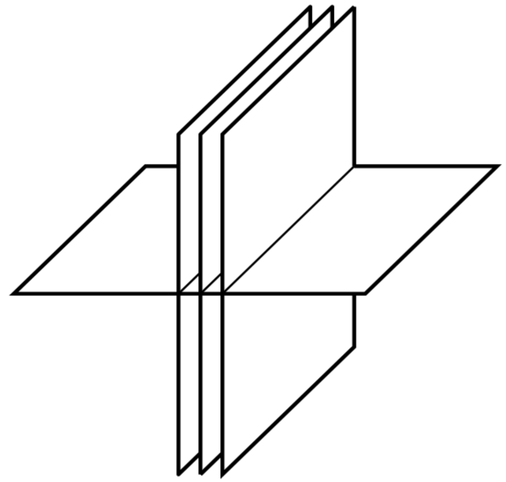
k charge units on the defect

	0	1	2	3	4	5	6	7	8	9
$N\ D3$	X	X	X	X						
$D5$	X					X	X	X	X	X
$k\ F1$	X				X					

$g_s N \gg 1$

$g_s N \ll 1$

Closed String Description



Open String Description

Maldacena Limit

Strings on $AdS_5 \times S^5$

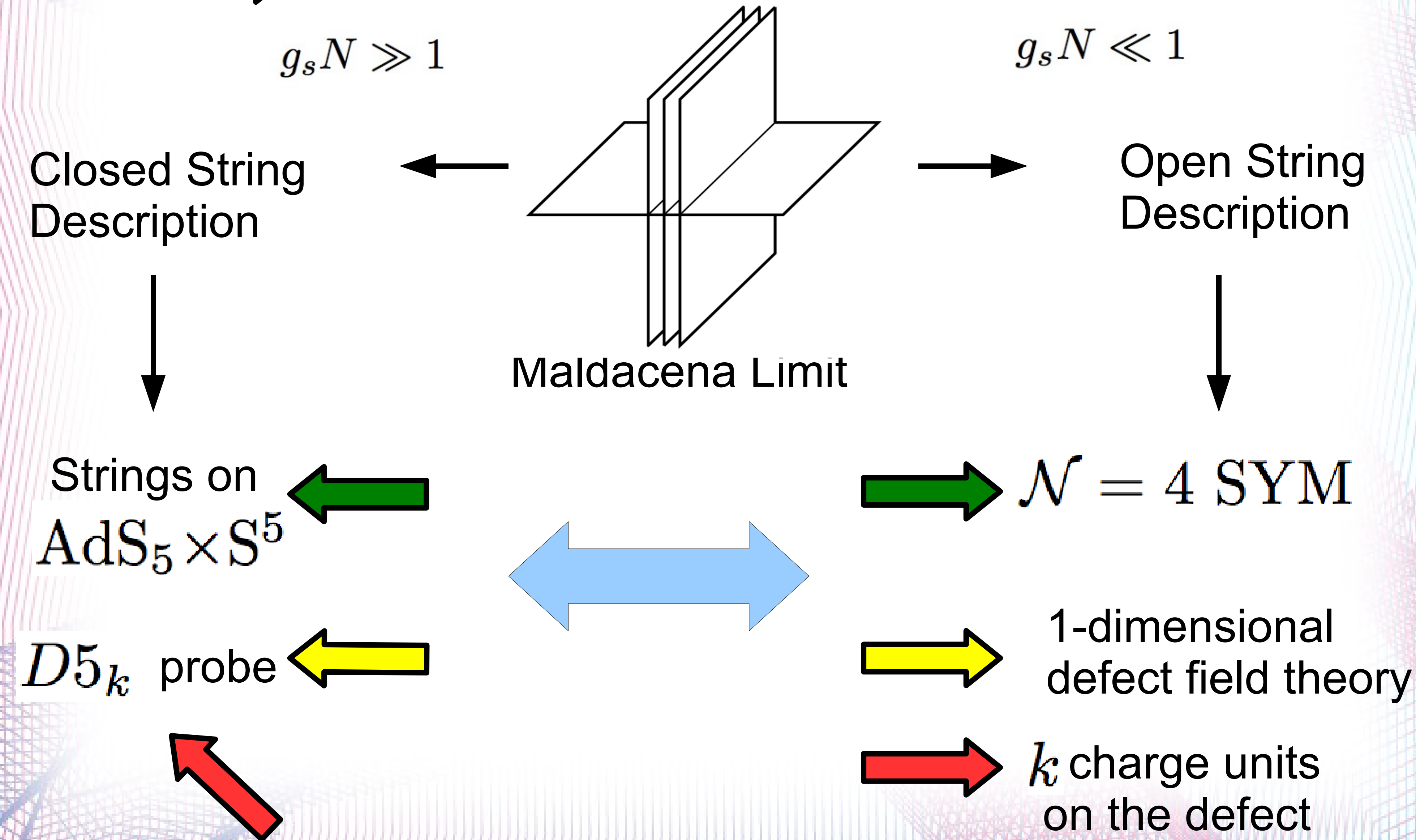
$\mathcal{N} = 4$ SYM

$D5_k$ probe

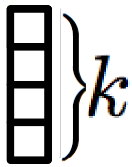
1-dimensional defect field theory

k charge units on the defect

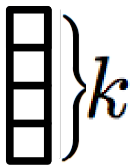
	0	1	2	3	4	5	6	7	8	9
$N\ D3$	X	X	X	X						
$D5$	X					X	X	X	X	X
$k\ F1$	X				X					



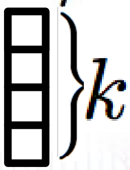
- Integrating out the d.o.f. localized on the defect, insert in the theory a Wilson loop W



- Thus $D5_k \Longleftrightarrow W$



and at strong coupling $\langle W \rangle \sim e^{-S_{D5_k}}$

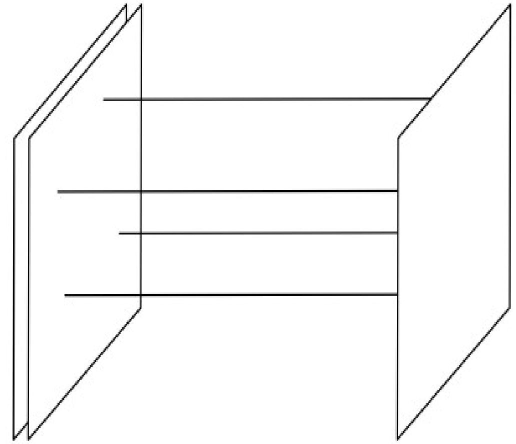


	0	1	2	3	4	5	6	7	8	9
$N\ D3$	X	X	X	X						
$D3$	X	X	X	X						
$k\ F1$	X				X					

$g_s N \gg 1$

$g_s N \ll 1$

Closed String Description



Open String Description

Maldacena Limit

Strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

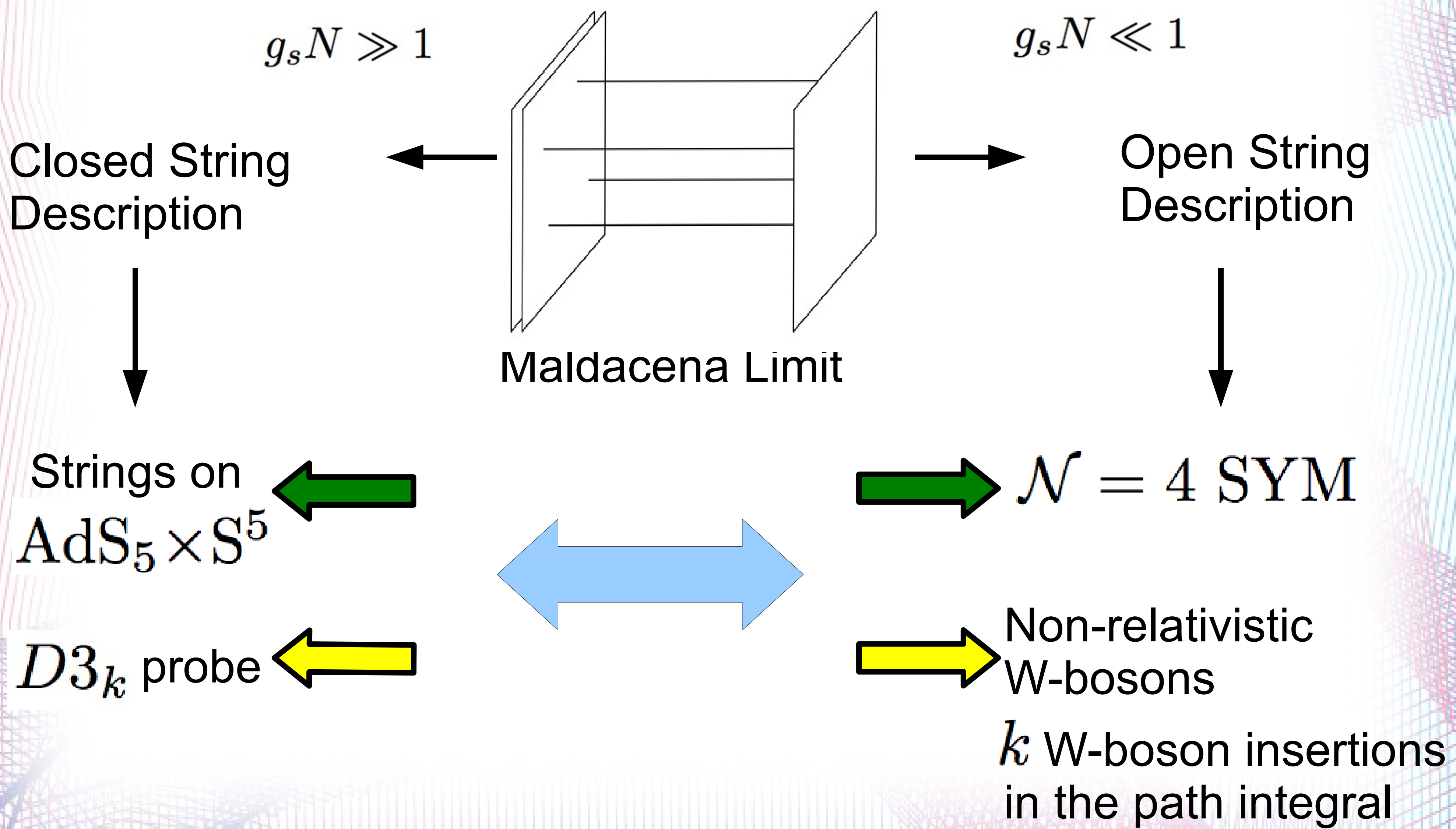
$\mathcal{N} = 4$ SYM

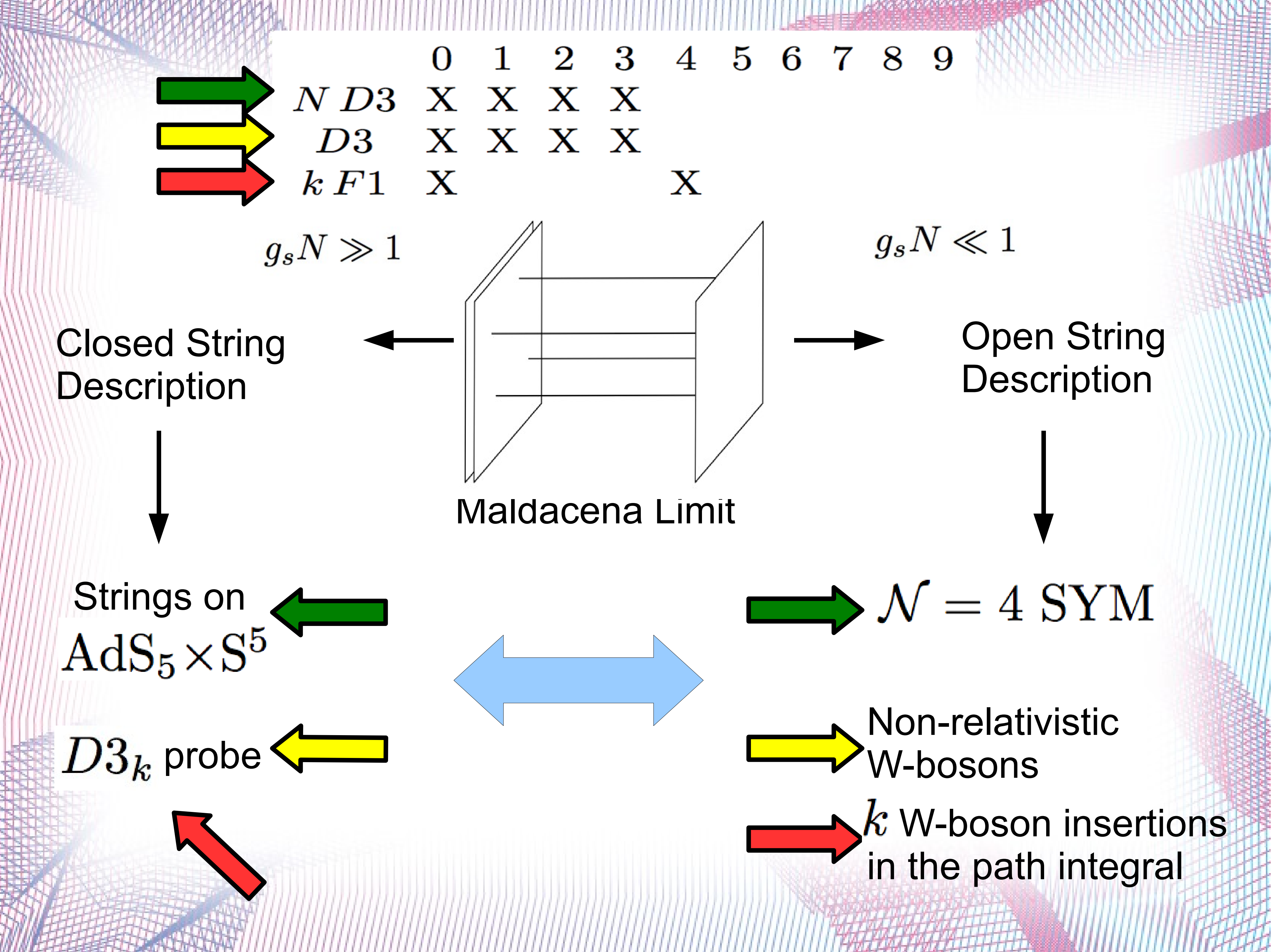
$D3_k$ probe

Non-relativistic W-bosons

k W-boson insertions in the path integral

	0	1	2	3	4	5	6	7	8	9
$N\ D3$	X	X	X	X						
$D3$	X	X	X	X						
$k\ F1$	X				X					





0 1 2 3 4 5 6 7 8 9
$N\ D3$ X X X X
$D3$ X X X X
$k\ F1$ X X
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Non-relativistic W-bosons
k W-boson insertions in the path integral

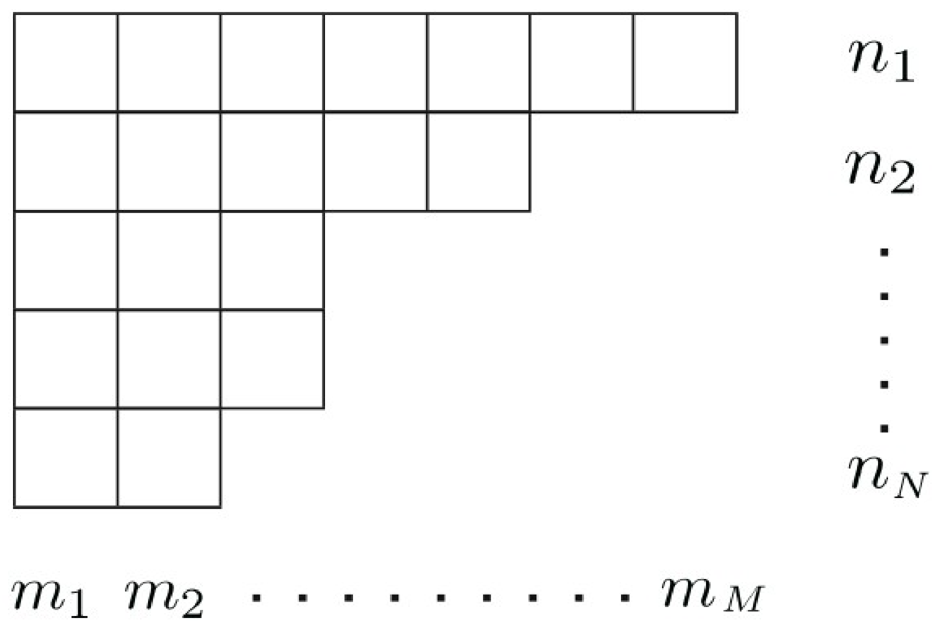
- Integrating out the d.o.f. associated to the W-bosons, insert in the theory a Wilson loop $W_{\underbrace{\square\square\square\square}_{k}}$

- Thus $$D3_k \Longleftrightarrow W_{\underbrace{\square\square\square\square}_{k}}$$

and at strong coupling $$\langle W_{\underbrace{\square\square\square\square}_{k}} \rangle \sim e^{-S_{D3_k}}$$

Generic Representation

- A Half-BPS W-L in a generic representation



is associated to $\left(D5_{m_1}, D5_{m_2}, \ldots, D5_{m_M}\right)$

or equivalently $\left(D3_{n_1}, D3_{n_2}, \ldots, D3_{n_N}\right)$

Conclusions

- Higher order representations W-L are described holographically by D-branes

  -This proposal has been verified considering the expectation value of circular W-L, providing a new non trivial check of the AdS/CFT correspondence

  - This analysis provides the string theory dual of all the gauge theory electric charges, extending in a significant way the AdS/CFT dictionary

  - New tool to compute the expectation value of higher order W-L at strong coupling

Conclusions

- The method developed can be used to study the AdS dual of other CFT operators
    - E.G. The surface operators [Chapter 3]